

# Applying Monte Carlo Simulations in Litigation

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In this article, the author illustrates how experts use Monte Carlo simulation to address a wide variety of issues that arise in litigation.

Anything that affects the value of an asset, even election results, weather conditions in a certain region, or results of a sporting event, can be converted into numbers and simulated. Simulation, which allows for the consideration of many potential future realizations of events, conditions, or payoffs, is a powerful tool. For some problems, for example, determining the value of a European call option, a mathematical formula can provide a theoretical solution. However, most real-world problems are so rich in detail that formulaic solutions either do not exist, or developing them is not practical.

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The use of simulation in solving such problems is not unique to economics and finance. Simulation is employed in many areas, including physics, chemistry, and even gaming. One of its most famous applications is Edward O. Thorp's study of optimal blackjack strategies. Ed Thorp was a mathematician at MIT in the late 1950s who studied card counting strategies and the associated probabilities of winning at blackjack. He used an IBM machine to calculate those probabilities by simulating game outcomes. This helped him develop optimal card counting strategies (he later wrote a *New York Times* best-selling book on card counting strategies, entitled *Beat the Dealer*). When he was ready to test his results, he went to Las Vegas and was so successful that casino security expelled him from the premises. Inspired by his success in gaming, Ed Thorp decided to employ his knowledge of statistics and simulation in the stock market and became a well-known hedge fund manager.

## MONTE CARLO SIMULATIONS IN FINANCE

Since those early days, simulation has become a standard tool on Wall Street. Its most standard application in finance is in modeling stock prices, typically referred to as Monte Carlo simulation. This simulation technique generates thousands of possible stock price paths that might be used, for example, for pricing complex derivatives. The price paths are generated by breaking the relevant time period into small intervals. The process starts with a known initial price, usually the stock price at a set time. Then small price increments (or stock returns) are selected (or "drawn") from a stochastic distribution one by one for each interval. Typically, the distribution from which interval returns are drawn is assumed to be a Normal distribution<sup>1</sup> with parameters informed by statistical properties of historical stock price movements.

In the next step of the analysis, interval returns are accumulated and applied to the initial stock price to construct a complete price path. Repeating this procedure thousands of times generates a distribution of price paths that can be used for calculating the expected returns and, based on the expected returns, the values of derivative instruments. It is advantageous to select the length of time intervals to be small (e.g., a day) and the number of simulations to be large (e.g., ten thousand).

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## ABOUT THE AUTHOR

**Steven R. Grenadier** is the William F. Sharpe Professor of Financial Economics at the Graduate School of Business at Stanford University. He specializes in complex financial asset analysis, and teaches courses on investments, portfolio management, and finance theory. Professor Grenadier has testified as an expert in investments and derivatives cases in which he has addressed valuation, disclosure, and suitability issues.

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1. Often changes in logarithms of prices are modeled instead of returns. In those cases the Log-normal distribution is typically used.

## MONTE CARLO SIMULATIONS IN LITIGATION

Monte Carlo simulations have a diverse set of applications in litigation. They include such broad issues as valuation, risk management, and calculating probabilities of various events. The following cases illustrate the power and flexibility of Monte Carlo simulations.

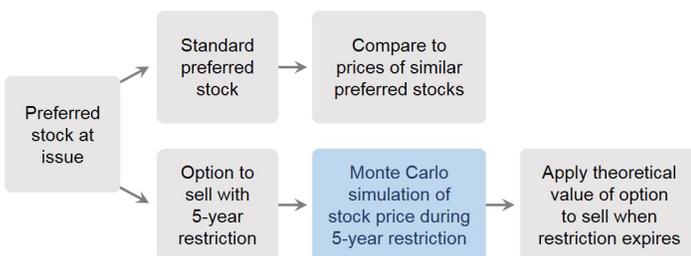
### Preferred Stock with Special Features

In a dispute that arose during an acquisition of a company, certain minority shareholders were not satisfied with the valuation of their claims and decided to litigate. The acquiring company offered these shareholders two options: either the cash value of their shares, which could trigger certain tax expenses, or a newly issued preferred stock, which would not.

The shareholders claimed that the value of the newly issued stock was substantially below the value of their previously held shares, and demanded compensation for the difference. Preferred stock is a hybrid security with cash flows more similar to those of a bond and with rights more similar to those of common stock, including voting rights. An expert was asked to assess the fair value of the preferred stock, which was complicated by the fact that the preferred shares had a number of unique features.

Since some of the features were not standard, the expert could not simply look up prices of similar securities. To address this the expert divided the preferred stock into two components. The first was a more standard preferred security, the value of which could be calculated based on prices of comparable securities. The second was the less standard feature: an option to sell the stock back to the company at any time after five years for a known cash value. The value of the preferred stock in question would be the sum of the values of these two components.

#### Overview of Valuation Steps



The expert calculated the value of the first component using prices of similar securities, that is, preferred stocks in the same industry with similar coupon payments and protections. In addition to prices, the expert also estimated the average yield of those securities, and their average volatility. These characteristics affected the distribution used in a Monte Carlo simulation of the option to sell (the second component).

To estimate the value of the second component (an option to sell the stock back to the company), the expert simulated the preferred stock's price path during the five-year period when the stock could not be sold back to the company and applied a known theoretical value of the option at the end of that period.

### Shareholders claimed that the value of a newly issued stock was substantially below the value of their previously held shares.

The mechanics of the Monte Carlo simulation are similar to the example provided in the introduction. Starting with the preferred stock's price on the valuation date, the expert drew random returns from a Normal distribution with parameters based on the risk-free rate, the average yield of comparable preferred stocks, and the average volatility of those yields. With a correct choice of distribution parameters, the simulation took into account the price of the risk embedded in the stock's price. For each sequence of returns the expert calculated the price path during the restriction period. At the end of the period the value of the option was determined using a known formula. This process was repeated thousands of times. In the end, the average option value across all simulations was calculated and discounted to the valuation date.

The option to sell the stock back to the company was valuable and ignoring it would not have yielded a fair estimate. Based on the expert's analysis, however, the expert was able to conclude that the valuation of the preferred stock was equivalent to the value of the cash offer.

Monte Carlo simulations can be used to value all kinds of securities. In this case, the most reliable results were obtained by combining a Monte Carlo simulation with market prices and formulaic solutions. When available, it is best to use market prices because they incorporate the views of a range of market participants. For components without market prices, however, a Monte Carlo simulation can provide a reasonable theoretical value.

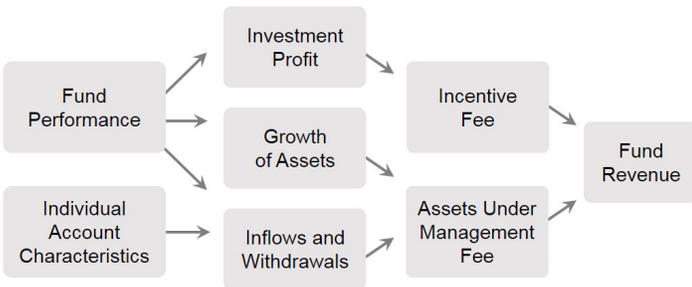
### Valuation of an Asset Management Firm

Monte Carlo simulations can be used not only to value financial derivatives or assess their risks, but to value companies, such as asset management firms, that have highly complex cash flow profiles. In a dispute over a hedge fund failure, an expert was asked to value the firm that managed the fund and assess how the firm’s value was affected by the fund’s performance.

#### Estimating the firm’s value involved simulating the fund’s performance as well as the inflows and withdrawals from the fund.

The value of a hedge fund management firm is based on its expected management and incentive fees. The management fee is usually a percentage of the value of assets under management, and the incentive fee is typically a percentage of the investment profits. Hence, the estimation of the firm’s value involved simulating the fund’s performance as well as the inflows and withdrawals from the fund, which also were affected by its performance.

#### Effects of Fund Performance and Individual Characteristics on Fund Review



To gather the appropriate information for the simulation, the expert investigated two items. First, based on historical data, the expert estimated how fund performance, age of individual accounts, and other characteristics affected inflows and withdrawals. Second, the expert reviewed the investment portfolio of the fund and its investment strategy. Based on the type of securities in which the fund invested, the expert determined the appropriate model for investment returns.

The starting point for the simulation was the amount of funds under management at the valuation date. Then the expert drew a possible performance path based on the fund’s investment strategy. For that performance path the expert estimated the inflows and withdrawals from the fund, the evolution of assets under management, and ultimately the flow of management and incentive fees to the management firm. This procedure was repeated thousands of times to evaluate how likely each of the outcomes was. Finally, the flow of fees was averaged across all simulations and discounted to the valuation date.

As this example shows, a Monte Carlo simulation is especially useful when the value of a firm can be viewed as comprising many separate accounts that can be modeled individually. This strategy can also be used, for example, for pools of mortgages. Performance of each mortgage can be modeled based on macroeconomic trends and individual account characteristics, for example, FICO scores and Loan-to-Value ratios. In turn, Monte Carlo simulation can be used to predict the performance and value of the entire pool.

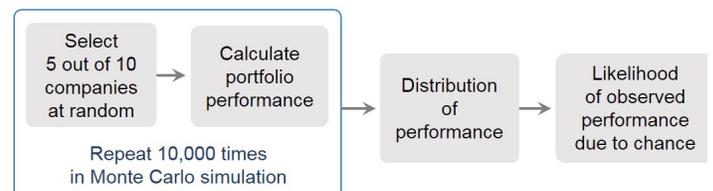
### The Effect of Selection Errors on Index Performance

Monte Carlo simulation can also be used to numerically evaluate how likely certain events might occur. In a dispute involving an investment portfolio, the parties disagreed about the potential effect of selection errors on portfolio performance.

The portfolio was selected by taking the top five stocks in each industry according to a particular criterion. The portfolio was back-tested and its back-tested performance was reported to investors. Later, certain errors in the testing process were uncovered, leading to a slightly different selection of companies than the criterion prescribed. The plaintiffs argued that such selection errors potentially could overstate performance and mislead investors. Thus, an expert was asked to evaluate how likely it was for the type of errors in question to lead to a performance overstatement.

To answer this question the expert used a Monte Carlo simulation. The expert opined that the effect of those errors at most would be equivalent to selecting five random companies out of the top ten companies ranked according to the correct selection criterion. The expert repeated the random selection process thousands of times and calculated performance for each selection. The expert then compared the simulated distribution to the actual performance and determined that only in less than 1 percent of cases the simulated performance was as high as or higher than the correct performance. Based on the simulation results, the expert concluded that errors in the selection process were highly unlikely to overstate portfolio performance.

#### Simulation of Likelihood of Portfolio Performance Due to Chance



**Risk of Collateralized Debt Obligation (CDO) Contracts**

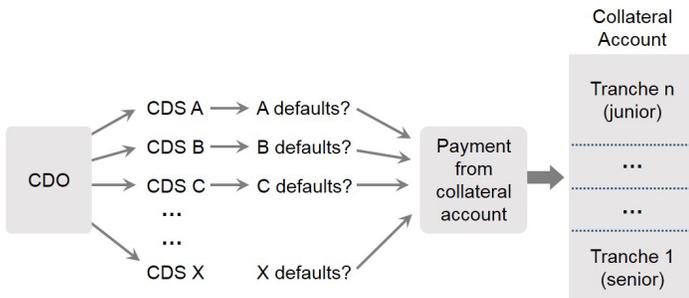
Risk management is another area where Monte Carlo simulation can be applied. In this case, the plaintiffs claimed that the risk of an investment—a synthetic CDO tranche, which became worthless as a result of the financial crisis—was misrepresented to them. An expert was asked to opine on the level of risk of the investment.

Synthetic CDOs are complicated derivative contracts whose value is based on a pool of credit default swap (CDS) contracts. A CDS allows for the transfer of an asset’s risk from one party to another, with the “insuring party” receiving a fixed periodic payment for the life of the agreement, and the “insured party” receiving a payment when a specified credit event occurs on the reference asset. Credit events are typically defined to include failure to make payments when due, bankruptcy, debt restructuring, a change in external credit rating, or rescheduling payments for the asset. The size of the payment is usually linked to the decline in the reference asset’s market value following the credit event.

Each investor in a synthetic CDO contributes collateral to an account that is later used for payments on individual CDSs when credit events occur. Typically, CDOs are structured in multiple tranches. Credit event payments are made first from the collateral account of the most junior tranche. Once collateral from that tranche is depleted, the next tranche is used and so on. Structuring CDOs in tranches allows investors to choose the level of risk they want to take. The most junior tranche has the most risk, and the most senior tranche has the lowest risk.

As one can imagine, estimating risk levels for each tranche is not a trivial task. One complication is the fact that defaults on reference assets are correlated. In unfavorable macroeconomic conditions, such as a financial crisis, multiple assets may default at the same time, while none may default in favorable macroeconomic conditions.

**Simulation of CDO Cash Flows**



Monte Carlo simulation is useful in assessing the risks of such a security. The key input parameters are default probabilities for the underlying reference assets (typically provided by rating agencies), losses given default (also provided by rating agencies), and joint probabilities of default, that is, how likely the reference assets are to default together (estimated in academic literature). These parameters determine the distribution from which defaults on reference assets can be simulated.

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**Plaintiffs claimed that the risk of an investment, which became worthless as a result of the financial crisis, was misrepresented to them.**

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The simulation starts with no assets in default. Then for each year the set of defaulted assets is drawn at random from the set of not yet defaulted assets in the pool. A draw is made in accordance with average default probabilities that correspond to asset ratings. Collateral balance in the CDS for each tranche is calculated accordingly. This procedure is repeated thousands of times to determine the proportion of times each tranche eventually gets depleted. These proportions constitute estimates of the probabilities of depletion for each tranche, which determines their risk.

The expert concluded that historical data predicted relatively low probability of loss at the time of investment.

**CONCLUDING REMARKS**

Monte Carlo simulation is a powerful tool that can be used to address a variety of questions that arise in litigation, the most common of which are related to valuation. For example, Monte Carlo simulations can be used to assess theoretical values of complicated financial instruments and companies. In addition to the examples in this article, areas of application include valuations of long-term commercial real estate leases, mortgage-backed securities, and many other instruments.

Another important application of simulation in litigation is risk assessment, as in the example of the synthetic CDO matter. Similarly, Monte Carlo simulations can be used to estimate the likelihood of different outcomes, such as an option being exercised or a portfolio yielding a certain return.

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